

Nonlinear field theories in inflationary scenarios

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Abstract

The effect of a uniform dilatation of space on stochastically driven nonlinear field theories is examined. This theoretical question serves as a model problem for examining the properties of nonlinear field theories embedded in expanding Euclidean Friedmann-Lemaître-Robertson-Walker metrics in the context of cosmology, as well as different systems in the disciplines of statistical mechanics and condensed matter physics. Field theories are characterized by the speed at which they propagate correlations within themselves. We show that for linear field theories correlations stop propagating if and only if the speed at which the space dilates is higher than the speed at which correlations propagate. The situation is in general different for nonlinear field theories. In this case correlations might stop propagating even if the velocity at which space dilates is lower than the velocity at which correlations propagate. In particular, these results imply that it is not possible to characterize the dynamics of a nonlinear field theory in an inflationary scenario *a priori*.

PACS numbers: 03.50.-z, 64.60.Ht, 89.75.Da, 98.80.Cq

Inflation is a cosmological concept that refers to the exponentially fast growth of the universe at its early stages. This way, inflation would be responsible for the current homogeneous and isotropic appearance of the universe. In this work we will use the term in a broader sense. We will consider stochastic field theories whose correlation length can be explicitly computed. When these theories are embedded in an universe which is uniformly expanding in time, what mathematically reduces to considering a Friedmann-Lemaître-Robertson-Walker (FLRW) metric, an effective loss of correlation takes place whenever the expansion is fast enough. Intuitively, determining the threshold of the speed of expansion that causes this loss of correlation would mean to find the condition that ensures the macroscopic appearance of the universe is homogeneous and isotropic. Although in reality this condition is just a necessary but not sufficient one, it is still one of the key ingredients in the search for determining the macroscopic appearance of one such universe. This problem has also a very natural statistical mechanical motivation. First of all, the models we are going to consider are paradigmatic in the theory of dynamic critical phenomena [1, 2]. In this context one would like to determine whether or not fluctuations are able to break the homogeneity of a space which undergoes a uniform dilatation, given that this mechanism acts as a homogenization on the large scale. On one hand, this question is clearly reminiscent of symmetry breaking problems in the context of nonequilibrium phase transitions. On the other hand, biological systems have a connection with it too, mainly in the context of pattern formation and generation of form during growth [3, 4]. Indeed, the equations this work is focused on can be considered as hydrodynamic descriptions of the model Eden introduced as an idealized description of a growing cell colony [5, 6].

In this work we are concerned with stochastic fields $\phi(x, t)$ for which the spatial coordinate is vectorial $x \in \mathbb{R}^d$ and the temporal coordinate is scalar $t \in \mathbb{R}^+$; in fact we will consider an origin of time $t_0 > 0$ so t is necessarily equal or greater than t_0 . This field will obey the generic equation of motion

$$\partial_t \phi = f(\hat{L}_1 \phi, \hat{L}_2 \phi, \dots) + \xi(x, t), \quad (1)$$

where the \hat{L}_i 's, $i = 1, 2, \dots$, are linear differential or integro-differential operators acting on the field, f is an in general nonlinear function of its arguments and ξ denotes a space-time noise to be specified in the following.

Field theories can be characterized by sets of exponents. One of them is the dynamic

exponent z which characterizes the velocity at which correlations propagate. If ℓ is the correlation length of one such theory then $\ell(t) \sim t^{1/z}$; therefore $z > 0$ so correlations propagate as time evolves.

We will consider inflation as a simple transformation of the spatial coordinates $x \rightarrow b(t)x$, where $b(t)$ is a function of the temporal variable only such that $b(t) > 1$ for $t > t_0$ and $b(t_0) = 1$. So this transformation is a strict dilatation.

Field theories can be characterized by different types of correlation functions, such as the two-point function and the field difference correlation function. For the models under consideration both will undergo dynamic scale invariance; so explicitly they read, the first one

$$G_2(x, x'; t)^2 := \langle \phi(x, t) \phi(x', t) \rangle = |x - x'|^{2\alpha} \mathcal{F} \left(\frac{|x - x'|}{t^{1/z}} \right), \quad (2)$$

and the second one

$$G_d(x, x'; t)^2 := \langle |\phi(x, t) - \phi(x', t)|^2 \rangle = |x - x'|^{2\alpha} \mathcal{G} \left(\frac{|x - x'|}{t^{1/z}} \right), \quad (3)$$

where role of the dynamic exponent is evident and the α exponent describes the variation of the field on a determined length scale [7]; \mathcal{F}, \mathcal{G} are the scaling functions.

We say that a *linear superposition principle* holds whenever, upon applying the inflation transformation $\{x, x'\} \rightarrow b(t)\{x, x'\}$, the correlation functions read

$$G_{\{2,d\}}^2 = b(t)^{2\alpha} |x - x'|^{2\alpha} \{\mathcal{F}, \mathcal{G}\} \left(\frac{b(t)|x - x'|}{t^{1/z}} \right), \quad (4)$$

for the *same* exponents α and z . It is clear where this expression comes from: in this case the internal dynamics of the field and the inflationary effect are simply superposed.

We start our discussion with the following family of linear equations of motion for the field ϕ :

$$\partial_t \phi = -\nu |\nabla|^\zeta \phi + h(t) + \xi(x, t), \quad (5)$$

where the noise is assumed to be Gaussian and white, with zero mean and correlation

$$\langle \xi(x, t) \xi(x', t') \rangle = D \delta(x - x') \delta(t - t'), \quad (6)$$

and the operator $|\nabla|^\zeta$ (we will always consider $\zeta > 0$) is to be interpreted in the Fourier transform sense ($|\nabla|^\zeta \phi = |k|^\zeta \hat{\phi}$). This operator accounts for the anomalous diffusion of the field, and its effect on this type of theories has already been considered, even in the

nonlinear case [8]. First we note that the appearance of the function $h(t)$ is trivial as it can be adsorbed by means of the transformation $\phi \rightarrow \phi + \int h(t)dt$ [9], and we will do so in the following. This linear field theory can be exactly integrated and shown to obey the above mentioned scalings with $z = \zeta$ (note that for ζ integer and even the operator becomes, up to its sign, the Laplacian or a power of it). Actually, the presence of noise in this equation is trivial in the sense that the exponent z does not change if we set $D = 0$ (trivial in this respect, not necessarily in others).

Now we will apply the inflation transformation to Eq. (5), and for the sake of concreteness we set $b(t) = (t/t_0)^\gamma$, for $\gamma > 0$, and we will refer to this parameter as the *growth index*. The natural question that arises is: is there linear superposition for this case? Even in this case in which the equation is linear the answer is only partially positive (and so in general it is negative). Linear superposition holds for both correlation functions, without amendments, only if $\gamma < 1/\zeta$. For $\gamma > 1/\zeta$ the two-point function does not adopt the form described by Eq. (4) [4, 10–12]. Indeed, this value of γ plays a special role. For $\gamma < 1/\zeta$ one can read from Eq. (4) that correlations still propagate as time evolves. The contrary would happen if we reversed the inequality. So the linear superposition principle has a transparent physical meaning: propagation of correlations and dilatation of space are simply superposed. Although linear superposition does not take place for the two point function and large enough γ , we still have a weaker yet intuitive result: propagation of correlations stops (or at most becomes logarithmically slow) in the limit $\gamma \rightarrow (1/\zeta)^-$ generically [11]; in the general case we should say that propagation of correlations stops whenever $\gamma > 1/\zeta$, as some particular cases admit a different marginal propagation of correlations for $\gamma = 1/\zeta$ [12, 13].

Explicitly, after applying the inflation transformation, the equation of motion reads

$$\partial_t \phi = - \left(\frac{t_0}{t} \right)^{\zeta \gamma} \nu |\nabla|^\zeta \phi + \left(\frac{t_0}{t} \right)^{d\gamma/2} \xi(x, t). \quad (7)$$

Following Eq. (4) and for $\gamma < 1/\zeta$ one may define an effective dynamic exponent $z_{\text{eff}} = \zeta/(1 - \gamma\zeta)$ [11]. Thus, in particular, in the limit $\gamma \rightarrow 0^+$ one recovers the classical case $z_{\text{eff}} \rightarrow \zeta$, and when $\gamma \rightarrow (1/\zeta)^-$ then $z_{\text{eff}} \rightarrow \infty$. So we may talk about the decorrelation threshold $\gamma_d = 1/\zeta$. As we have already mentioned, although linear superposition does not completely hold in this case, the decorrelation threshold is the same as if it held.

The main conclusion of this analysis is that for a rather general family of linear equations the decorrelation threshold is the intuitive one. We will show that for nonlinear equations

things are in general different. To this end one needs to introduce a nonlinear field theory whose dynamic exponent is nontrivial. One such theory is given by the Kardar-Parisi-Zhang (KPZ) equation [14]

$$\partial_t \phi = \nu \nabla^2 \phi + \frac{\lambda}{2} (\nabla \phi)^2 + \xi(x, t). \quad (8)$$

Together with the interest of this equation in the fields of condensed matter and statistical physics [15], one finds its relevance in cosmology and related topics [16–24]. Two of the reasons underlying this universal character are the connection of Eq. (8) with Burgers equation through the definition of the velocity field $\mathbf{v} := \nabla \phi$ and to the imaginary time Schrödinger equation with a random potential by means of the change of variables $\psi := \exp[\lambda \phi / (2\nu)]$. If we set $D = 0$ in this equation we find that $z = 2$, as can be read from its exact solution in this case [14], or even by simple dimensional analysis. However, contrary to what happened to the linear equations, once the noise is switched on, the dynamic exponent becomes a function of the spatial dimension, $z = z(d)$. In particular, it is known that $z(1) = 3/2$ and $z(2) \approx 1.7 < 2$. We note that while the one-dimensional result is exact, the two-dimensional one is usually obtained with numerical methods. This is so because the calculation of this value has escaped all sorts of analytical approaches, with the notable exception of an improved-renormalization-group-type method known as the Self-Consistent Expansion (SCE) [25, 26]. Due to the remarkable success of this scheme in finding the scaling behavior of this as well as different models [27–29] we will rely on its results in the following.

Our aim is calculating the decorrelation threshold for the KPZ equation. Following the linear theory one could naïvely expect $\gamma_d = 1/z$, and in particular $\gamma_d = 2/3$ in $d = 1$. In fact, the one dimensional result is presumably correct. Simulations of a discrete model in the KPZ universality class have corroborated this result [30]. Things are, on the other hand, different in higher dimensions as will shown in the forthcoming analysis.

We now apply to Eq. (8) the dilatation transformation $x \rightarrow (t/t_0)^\gamma x$, where as always $\gamma > 0$. The resulting equation reads

$$\partial_t \phi = \nu \left(\frac{t_0}{t} \right)^{2\gamma} \nabla^2 \phi + \frac{\lambda}{2} \left(\frac{t_0}{t} \right)^{2\gamma} (\nabla \phi)^2 + \left(\frac{t_0}{t} \right)^{d\gamma/2} \xi(x, t). \quad (9)$$

So we will study this equation which describes the KPZ dynamics in an environment which is *undergoing spatial dilatation* as time evolves. This is a Langevin equation whose associated

Fokker-Planck equation reads

$$\partial_t \mathcal{P} = \left(\frac{t_0}{t}\right)^{2\gamma} \int dx \frac{\delta}{\delta \phi} \left[\nu \nabla^2 \phi + \frac{\lambda}{2} (\nabla \phi)^2 \right] \mathcal{P} + \frac{D}{2} \left(\frac{t_0}{t}\right)^{d\gamma} \int dx \frac{\delta^2}{\delta \phi^2} \mathcal{P}, \quad (10)$$

where the solution \mathcal{P} is the functional probability distribution. This equation can be transformed to

$$\frac{\partial \mathcal{P}}{\left(\frac{t_0}{t}\right)^{2\gamma} \partial t} = \int dx \frac{\delta}{\delta \phi} \left[\nu \nabla^2 \phi + \frac{\lambda}{2} (\nabla \phi)^2 \right] \mathcal{P} + \frac{D}{2} \left(\frac{t_0}{t}\right)^{(d-2)\gamma} \int dx \frac{\delta^2}{\delta \phi^2} \mathcal{P}. \quad (11)$$

Now we change the temporal variable

$$d\tau = \left(\frac{t_0}{t}\right)^{2\gamma} dt \longrightarrow \tau = \frac{t_0^{2\gamma}}{1-2\gamma} (t^{1-2\gamma} - t_0^{1-2\gamma}). \quad (12)$$

We start assuming $\gamma < 1/2$ so that τ is growing for growing t and

$$\tau \approx \frac{t_0^{2\gamma}}{1-2\gamma} t^{1-2\gamma} \quad \text{when } t \rightarrow \infty. \quad (13)$$

After performing the change of variables and going back to the Langevin equation we find

$$\partial_\tau \phi = \nu \nabla^2 \phi + \frac{\lambda}{2} (\nabla \phi)^2 + \left(\frac{1-2\gamma}{t_0}\right)^{\frac{(2-d)\gamma}{2(1-2\gamma)}} \tau^{\frac{(2-d)\gamma}{2(1-2\gamma)}} \xi(x, \tau), \quad (14)$$

or alternatively

$$\partial_\tau \phi = \nu \nabla^2 \phi + \frac{\lambda}{2} (\nabla \phi)^2 + \eta(x, \tau), \quad (15)$$

where the noise correlations are $\langle \eta(x, \tau) \rangle = 0$ and

$$\langle \eta(x, \tau) \eta(x', \tau') \rangle = \tilde{D} \tau^{\frac{(2-d)\gamma}{1-2\gamma}} \delta(x - x') \delta(\tau - \tau'), \quad (16)$$

for a suitable constant \tilde{D} . This noise renders in general a more difficult analytical treatment due to the explicit τ -dependence of its amplitude. However, the situation becomes considerably simpler in $d = 2$. In this case we have

$$\partial_\tau \phi = \nu \nabla^2 \phi + \frac{\lambda}{2} (\nabla \phi)^2 + \xi(x, \tau), \quad (17)$$

which is the KPZ equation for time τ , so the dynamic exponent for this time variable is $z' = z_{\text{KPZ}}(d = 2)$. Thus the effective dynamic exponent for actual time t is

$$z_{\text{eff}} = \frac{z_{\text{KPZ}}(d = 2)}{1 - 2\gamma}. \quad (18)$$

Consequently decorrelation appears in the limit $\gamma \rightarrow (1/2)^- \Rightarrow \gamma_d = 1/2$, instead of $\gamma_d = 1/z_{\text{KPZ}}(d = 2) \approx 0.6 > 1/2$. So the decorrelation threshold is anticipated, and this counterintuitive result implies that a simple superposition principle does not hold in this case.

Although this result proves the decorrelation threshold by itself, it is easy to compute the exact behavior at the value $\gamma = 1/2$. In this case one can correspondingly modify change of variables (12) to find $\tau = t_0 \ln(t/t_0)$. So for the critical value of γ correlations propagate logarithmically slow, and we find the effective value $z_{\text{eff}} = \infty$. For $\gamma > 1/2$ change of variables (12) is still valid. One again finds that the solution of Eq. (9) becomes the solution of the classical KPZ equation in time τ . The particularity of this situation is that, as time t progresses, time τ evolves from $\tau = 0$ when $t = t_0$ to the finite value $\tau = t_0/(2\gamma - 1)$ in the limit $t \rightarrow \infty$. So the resulting profile of the solution to Eq. (9) becomes the profile of the solution to Eq. (8) quenched at time $t_0/(2\gamma - 1)$ asymptotically in time. This result, among other things, implies that correlations propagate for short times but tend to stop asymptotically in time. This means that the dilatation of space acts as an effective drift that is able to transport correlations by itself [11].

This effect is not purely two-dimensional. We now move to higher dimensions and consider again the KPZ equation but with a different stochastic forcing

$$\partial_t \phi = \nu \nabla^2 \phi + \frac{\lambda}{2} (\nabla \phi)^2 + \chi(x, t), \quad (19)$$

where the noise is assumed to be Gaussian, white in time and spatially correlated. We consider it has zero mean and explicitly its correlation reads

$$\langle \chi(x, t) \chi(x', t') \rangle = D |x - x'|^{2\rho-d} \delta(t - t'), \quad (20)$$

where $\rho > 0$ specifies the degree of spatial correlation ($\rho = 0$ sends us back to the spatially uncorrelated noise). This equation can again be mapped onto a Fokker-Planck description. And again, the same transformation $\tau = t_0^{2\gamma} (t^{1-2\gamma} - t_0^{1-2\gamma}) / (1 - 2\gamma)$, yields in the limit $t \rightarrow \infty$ the equation

$$\partial_\tau \phi = \nu \nabla^2 \phi + \frac{\lambda}{2} (\nabla \phi)^2 + \left(\frac{1 - 2\gamma}{t_0} \right)^{\frac{(2+2\rho-d)\gamma}{2(1-2\gamma)}} \tau^{\frac{(2+2\rho-d)\gamma}{2(1-2\gamma)}} \chi(x, \tau). \quad (21)$$

In this case this model becomes exactly Eq. (19) in time τ for $d = 2 + 2\rho$. So choosing appropriate integer and semi-integer values of ρ one recovers the KPZ equation in τ time

for any desired spatial dimension $d > 2$. Model (19) was analyzed with the SCE and for the dimension under examination $d = 2 + 2\rho$ classical KPZ behavior, as if $\rho = 0$, was found [31]. It is not clear whether or not there exist an upper critical dimension for KPZ (a dimension above which the large-scale effective behavior of the equation would reduce to that of its linear counterpart) and what would be its value in the first case [32, 33]. Recent numerical results suggest that, if it exist, one necessarily has $d_c > 4$ [34]. In any case, it is clear that for any $d \geq 2$ under the upper critical dimension of KPZ the corresponding dynamic exponent of model (19) is $z < 2$, while the decorrelation threshold is as before anticipated and results $\gamma_d = 1/2$. This proves that the nontrivial coupling of the inflationary transformation and the nonlinear field theory extends from two to higher dimensions, at least in the range $2 \leq d \leq 4$ according to [34], and possibly to higher dimensions. This also shows that apparently the one-dimensional situation is left alone as the only one in which the decorrelation threshold is the intuitive one. And thus, this fact is yet another proof of the fundamentally different character of the KPZ equation in and above one dimension, posing, in the latter case, a problem much more involved and changeling. This difference should be even more pronounced in the neighboring field of radial growth, since posing the KPZ problem in this context implies nontrivial topological effects only if $d \geq 2$ [4, 12].

As already mentioned in the introduction, the condition $\gamma > 1/z$ implies correlations stop propagating in the linear case, but it is just a necessary and not sufficient condition to achieve the spatial homogeneity of the field. Homogeneity is only achieved in the large scale if $\gamma > \max\{1/z, 1/d\}$ in the linear case [4], what shows that the spatial dimensionality of the system has an important role in this question. Although it would be tempting to simply extend the previous results concerning the linear models to the general case, we are going to show that such an extension is not correct. However, we will be able to find an analogous condition by means of the introduction of new critical exponents $\tilde{\alpha}$ and \tilde{z} ; furthermore, these new exponents will allow expressing the condition for decorrelation as $\gamma > 1/\tilde{z}$. To this end we will make explicit use of correlations (2) and (3) for the KPZ case. If we write these correlations in the form suggested by Eq. (4), so that the dependence on the dilatation of space becomes explicit, we find the expression

$$G_{\{2,d\}}^2 = t^{2\tilde{\alpha}\gamma} |x - x'|^{2\tilde{\alpha}} \left\{ \tilde{\mathcal{F}}, \tilde{\mathcal{G}} \right\} \left(\frac{t^\gamma |x - x'|}{t^{1/\tilde{z}}} \right), \quad (22)$$

where the new exponent values are $\tilde{\alpha} = (1 - 2\gamma)\alpha_{\text{KPZ}}/(1 - 2\gamma + z_{\text{KPZ}}\gamma)$ and $\tilde{z} = z_{\text{KPZ}}/(1 -$

$2\gamma + z_{\text{KPZ}}\gamma$) whenever $\gamma < 1/2$, where α_{KPZ} and z_{KPZ} are the corresponding exponents of KPZ at the lower critical dimension $d = 2 + 2\rho$ (including the case $\rho = 0$). For $\gamma \geq 1/2$ the exponents become $\tilde{\alpha} = 0$ and $\tilde{z} = 2$, with marginal logarithmic corrections for $\gamma = 1/2$. Also, if one uses the form of both correlations $G = t^{2\beta} \mathcal{H}(|x - x'|/t^{1/z})$ one may extract in our case the value of the new growth exponent $\tilde{\beta} = (1 - 2\gamma)\beta_{\text{KPZ}}$ for $\gamma < 1/2$ and $\tilde{\beta} = 0$ for $\gamma \geq 1/2$ (with again a marginal logarithmic correction for $\gamma = 1/2$) and find the relation $\tilde{\beta} = \tilde{\alpha}/\tilde{z}$ holds in this case too. Additionally one sees that using the relation $\alpha_{\text{KPZ}} + z_{\text{KPZ}} = 2$ one finds for the new exponents $\tilde{\alpha} + \tilde{z} = 2$. The different relation found for the KPZ equation with a time dependent coefficient of the nonlinearity [35] can be written in the present terms as $\alpha_{\text{KPZ}} + z_{\text{eff}} = 2 + 2\gamma z_{\text{eff}}$. From these results one reads that the dilatation of space systematically neglects the non-perturbational behavior of KPZ. A new quantity within reach is the center of mass fluctuations, which can be characterized by a new exponent $\int \phi(x, t) dx \sim t^\mu$ (this time the integral should be carried out on a bounded domain for obvious reasons). In the present case this exponent reads $\mu = [\beta + (d - 2\rho)/(2z)](1 - 2\gamma)$ for $\gamma < 1/2$ and $\mu = 0$ for $\gamma \geq 1/2$, with as previously a logarithmic correction for $\gamma = 1/2$. In particular, the motion of the center of mass becomes bounded in time for $\gamma > 1/2$. Note that this quantity is closely related to the properties characterizing weak convergence of the profile of ϕ to the homogeneous spatial state [4]. In general the exponents which characterize weak convergence to the homogeneous profile are different from the exponents appearing in the field difference correlation function (3) [4]. However, in the present cases, both sets of exponents are exactly the same. This should be considered by no means a general feature of the KPZ equation: it is a direct consequence of fact that we are always considering this equation at its lower critical dimension. Note also that our results imply the flatness of the field (in the sense that both field difference and two-point correlation functions are uniformly bounded in both space and time) is achieved for $\gamma > 1/2$. This is in disagreement with the linear requirement $\gamma > \max\{1/z, 1/d\}$. However, it would be in perfect agreement with the modified requirement $\gamma > \max\{1/\tilde{z}, 1/d\}$. In the same way, the threshold for the loss of correlation could be expressed by the inequality $\gamma > 1/\tilde{z}$. Both inequalities express the fact that the linear conditions for decorrelation and homogeneity of the field can be extended to the nonlinear case provided we introduce γ -dependent exponents. This is another way of expressing that the coupling of the inflation transformation and the nonlinear dynamics of the field is nontrivial.

We also note that, although we have proven the threshold $\gamma = 1/2$ for decorrelation and homogeneity of the field for the lower critical dimension of KPZ, we expect it will stay the same for dimensions above this one. This conjecture comes from the fact that change of variables (12) sends the equations under consideration to KPZ equations with weaker noises (noises whose amplitude depend on a negative power of time) in the case of a higher dimensionality. The question of *super-roughness* of the field, i. e. finding the values of γ for which the fluctuations of the field grow faster than the dilatation of space, is a simple corollary of our results. This would happen whenever $\tilde{\alpha} > 1$, what is impossible for any $\gamma \geq 0$.

In summary, we have studied the effect of a uniform dilatation of space, what we have refer to as the inflation transformation, on the dynamics of nonlinear fields theories. In particular we have focused on the nonlinear KPZ equation with different stochastic forcing terms, because this field theory is known to display nontrivial effects regarding the velocity at which correlations propagate. We have argued that in one dimension numerical results suggest that the loss of correlation starts when the velocity at which the space grows overtakes the velocity at which correlations propagate in the absence of inflation. However, in two and higher dimensions the threshold for the appearance of decorrelation becomes anticipated, and so loss of correlation starts at a velocity of the inflation transformation slower than the speed at which correlations propagate. This fact is a consequence of the nontrivial behavior of the KPZ equation at the lower critical dimension. It shows that the interplay of inflation and nonlinearity is far from trivial and, in particular, that it is not possible to infer the effect of a dilatation of space on a nonlinear field theory *a priori*.

There are several interesting connections among models in condensed matter physics and cosmology. In this work we have discussed one such model given by the KPZ equation, which lies in the mentioned interface as well as Ginzburg-Landau theories [36] and Bose-Einstein condensation [37]. Some questions naturally emerge from the present study. One is determining under which conditions loss of correlation in an anisotropically expanding universe is achieved. Mathematically, accounting for anisotropic expansions implies the substitution of the FLRW metric by a Bianchi I metric [38]. Another problem is the analysis of related nonlinear models with a source of quantum fluctuations instead of the classical ones. In this framework the question of under which conditions disentanglement occurs [39] seems to be connected with the present discussion.

This work has been partially supported by projects MTM2010-18128, RYC-2011-09025 and SEV-2011-0087.

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- [1] E. Katzav and M. Schwartz, EPL **95**, 66003 (2011).
 - [2] E. Katzav and M. Schwartz, Phys. Rev. Lett. **107**, 125701 (2011).
 - [3] C. Escudero, Arbor **186**, 1065 (2010).
 - [4] C. Escudero, Chaos, Solitons & Fractals **45**, 109 (2012).
 - [5] M. Eden, in *Symposium on Information Theory in Biology*, edited by H. P. Yockey (Pergamon Press, New York, 1958).
 - [6] M. Eden, in *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability*, edited by J. Neyman (University of California Press, Berkeley, 1961).
 - [7] The exponent α might indeed depend on the scale $|x - x'|$ if the variations of the field are different in different scales. Herein we will consider a constant α because we will focus always on the largest spatial scale.
 - [8] E. Katzav, Phys. Rev. E **68**, 031607 (2003).
 - [9] This function $h(t)$ can thought of as a time dependent mean of the noise term. In the context of the nonequilibrium mechanics of stochastic growth $h(t) = \gamma Ft^{\gamma-1}$, what means that the substrate is growing at a rate Ft^γ . Although its introduction is trivial in the present context, we have mantained its presence following the tradition in this field.
 - [10] C. Escudero, Phys. Rev. Lett. **100**, 116101 (2008).
 - [11] C. Escudero, J. Stat. Mech. P07020 (2009).
 - [12] C. Escudero, Phys. Rev. E **84**, 031131 (2011).
 - [13] C. Escudero, Ann. Phys. **324**, 1796 (2009).
 - [14] M. Kardar, G. Parisi, and Y.-C. Zhang, Phys. Rev. Lett. **56**, 889 (1986).
 - [15] H. S. Wio, C. Escudero, J. A. Revelli, R. R. Deza, and M. S. de La Lama, Phil. Trans. R. Soc. A **369**, 396 (2011).
 - [16] A. Berera and L.-Z. Fang, Phys. Rev. Lett. **72**, 458 (1994).
 - [17] J. F. Barbero G., A. Domínguez, T. Goldman, and J. Pérez-Mercader, Europhys. Lett. **38**, 637 (1997).
 - [18] A. Domínguez, D. Hochberg, J. M. Martín-García, J. Pérez-Mercader, and L. S. Schulman,

- Astron. Astrophys. **344**, 27 (1999).
- [19] T. Buchert, A. Domínguez, and J. Pérez-Mercader, Astron. Astrophys. **349**, 343 (1999).
 - [20] B. J. T. Jones, Mon. Not. R. Astron. Soc. **307**, 376 (1999).
 - [21] P. Coles, Mon. Not. R. Astron. Soc. **330**, 421 (2002).
 - [22] S. Matarrese and R. Mohayaee, Mon. Not. R. Astron. Soc. **329**, 37 (2002).
 - [23] P.-H. Chavanis, Phys. Rev. D **84**, 063518 (2011).
 - [24] S. N. Gurbatov, A. I. Saichev, and S. F. Shandarin, Phys.-Usp. **55**, 223 (2012).
 - [25] M. Schwartz and S. F. Edwards, Europhys. Lett. **20**, 301 (1992).
 - [26] M. Schwartz and S. F. Edwards, Phys. Rev. E **57**, 5730 (1998).
 - [27] E. Katzav, Physica A **309**, 79 (2002).
 - [28] E. Katzav, Phys. Rev. E **65**, 032103 (2002).
 - [29] E. Katzav, Phys. Rev. E **68**, 046113 (2003).
 - [30] J. M. Pastor and J. Galeano, Central European J. Phys. **5**, 539 (2007).
 - [31] E. Katzav and M. Schwartz, Phys. Rev. E **60**, 5677 (1999).
 - [32] E. Perlsman and M. Schwartz, Physica A **234**, 523 (1996).
 - [33] E. Katzav and M. Schwartz, Physica A **309**, 69 (2002).
 - [34] M. Schwartz and E. Perlsman, Phys. Rev. E **85**, 050103(R) (2012).
 - [35] E. Hernández-García, T. Ala-Nissila, and M. Grant, Europhys. Lett. **21**, 401 (1993).
 - [36] D. Boyanovsky, C. Destri, H. J. de Vega, and N. G. Sánchez, Int. J. Mod. Phys. A **24**, 3669 (2009).
 - [37] P. Jain, S. Weinfurtner, M. Visser, and C. W. Gardiner, Phys. Rev. A **76**, 033616 (2007).
 - [38] J. A. R. Cembranos, C. Hallabrin, A. L. Maroto, and S. J. Núñez Jareño, Phys. Rev. D **86**, 021301(R) (2012).
 - [39] Y. Nambu and Y. Ohsumi, Phys. Rev. D **80**, 124031 (2009).